UNIVERSITY OF YORK

BA, BSc and MMath Examinations 2014
MATHEMATICS
Calculus

Time Allowed: $1\frac{1}{2}$ hours.

- *Answer all questions.*
- *When answering a question you* ***must*** *use pencil, not ink and you should fully shade the box corresponding to your chosen answer.*
- *Circling, dotting or crossing the answer is not allowed.*
- *If you want to change your answer you must completely erase your incorrect mark using a rubber.*
- *Please use this question paper for all your rough work. The back of each page is blank for this purpose.*
- *Marks made on the exam paper will not count.*
- *Standard calculators will be provided.*

There are 25 *questions on this paper each worth* 1 *mark.*

1 (of 25). The derivative with respect to x of $x^{2\pi}$ is:

(A) $\frac{1}{2\pi}x^{2(\pi-1)}$ (B) $2\pi x^{2\pi-1}$ (C) $2\pi x^{2\pi+1}$ (D) $\frac{1}{2\pi}x^{2\pi+1}$
(E) None of these.

2 (of 25). The derivative with respect to x of $\coth 2x$, where $\coth y := 1/\tanh y$, is:

(A) $2\,\mathrm{sech}^2(2x)$ (B) $2 - 2\coth^2(2x)$
(C) $2\tanh(2x)\,\mathrm{sech}(2x)$ (D) $2\coth(2x)\,\mathrm{cosech}(2x)$
(E) None of these.

3 (of 25). The derivative with respect to x of $\cosh^{-1}(x)$ is:

(A) $\dfrac{1}{1+x^2}$ (B) $\dfrac{1}{\sqrt{1+x^2}}$
(C) $\dfrac{-1}{1-x^2}$ (D) $\dfrac{1}{\sqrt{x^2-1}}$
(E) None of these.

4 (of 25). The derivative with respect to x of 2^{3x} is:

(A) $2(3^{2x})$ (B) $3\ln(2)3^{2x}$ (C) $3\ln(2)2^{3x}$ (D) $2x3^{2x-1}$
(E) None of these.

5 (of 25). The derivative with respect to x of $\int_x^1 e^{t^2}\,dt$ is:

(A) $-e^{x^2}$ (B) e^{x^2} (C) e^{-x^2} (D) $2xe^{x^2}$
(E) None of these.

6 (of 25). The indefinite integral $\int \pi^x\,dx$ is given by:

(A) $\ln(\pi)\pi^x + C$ (B) $\ln(\pi)\pi^{x+1} + C$
(C) $\dfrac{\pi^x}{\ln(\pi)} + C$ (D) $\dfrac{\pi^{x+1}}{\pi+1} + C$
(E) None of these.

7 (of 25). The indefinite integral $\int xe^{x^2}\,dx$ is given by:

(A) $\dfrac{1}{2x}e^{x^2}+C$ (B) $e^{x^2}+C$
(C) $\frac{1}{2}e^{x^2+1}+C$ (D) $\frac{1}{2}e^{x^2}+C$
(E) None of these.

8 (of 25). The indefinite integral $\displaystyle\int \frac{2}{\sqrt{1-4x^2}}\,dx$ is given by:

(A) $2\arcsin(x)+C$ (B) $\arctan(2x)+C$
(C) $\arccos(x/2)+C$ (D) $\arcsin(2x)+C$
(E) None of these.

9 (of 25). The tangent line to the curve $y=x^x$ at $x=2$ has slope:

(A) $4+4\ln 2$ (B) $4\ln(2)$ (C) 3 (D) $2\ln(2)+2$
(E) None of these.

10 (of 25). The tangent line to the curve $ye^y=x^2$ at the point $(\sqrt{e},1)$ has slope:

(A) $\dfrac{2}{\sqrt{e}}$ (B) $\sqrt{e}$ (C) 1 (D) $\dfrac{1}{\sqrt{e}}$
(E) None of these.

11 (of 25). The integrating factor for the differential equation $y'+\dfrac{4}{x^2}y=\ln x$ is

(A) $\frac{-4}{x}$ (B) e^{4/x^2} (C) $\frac{4}{x^2}e^{-4/x}$ (D) e^{-2/x^3}
(E) None of these.

12 (of 25). The general solution to the differential equation $y'=3y\sqrt{x}$ is given by:

(A) $Ae^{2x^{3/2}}$ (B) $e^{Ax^{3/2}}$
(C) $e^{x^{3/2}}+A$ (D) $e^{x^{2/2}+A}$
(E) None of these.

where A is an arbitrary constant.

13 (of 25). The value of the improper integral $\displaystyle\int_{-3}^{0} \frac{1}{\sqrt{3+x}}\,dx$ is:

(A) $-2\sqrt{3}$ (B) undefined (C) $2\sqrt{3}$ (D) $\sqrt{3}$
(E) None of these.

14 (of 25). The value of the definite integral $\int_1^{2e} x^2 \ln x\,dx$ is:

(A) $\dfrac{(1-24\ln 2-16)e^3}{9}$ (B) $\dfrac{(1+24\ln 2+16)e^3}{9}$
(C) $\dfrac{1-(24\ln 2+16)e^3}{9}$ (D) $\dfrac{1+(24\ln 2+16)e^3}{9}$
(E) None of these.

15 (of 25). The derivative $f'(e)$ of the function $f(x)=\int_{-1}^{\ln x}\sqrt{3+t^4}\,dt$ is:

(A) $\dfrac{2}{e}$ (B) $\dfrac{\sqrt{2}}{2e}$ (C) $2\sqrt{e}$ (D) $\sqrt{\dfrac{2}{e}}$
(E) None of these.

16 (of 25). The value of the definite integral $\displaystyle\int_0^1 \frac{\arctan(x)}{1+x^2}\,dx$ is:

(A) $\dfrac{\pi^2}{16}$ (B) $\dfrac{\pi}{8}$ (C) $\dfrac{\pi^2}{32}$ (D) $\dfrac{\pi^2}{8}$
(E) None of these.

17 (of 25). The value of the improper integral $\displaystyle\int_1^{\infty} \frac{1}{(1+t)^{3/2}}\,dt$ is:

(A) $\frac{1}{2\sqrt{2}}$ (B) $-\frac{1}{2\sqrt{2}}$ (C) $\sqrt{2}$ (D) $-\sqrt{2}$
(E) None of these.

18 (of 25). The unique solution to the differential equation $y' + 3y = \sin(\pi x)$ with initial condition $y(0) = 1/2$ is:

(A) $y = e^{-3x}\left(\frac{1}{2} + \frac{\pi}{9+\pi^2}\right) - \frac{\pi\cos(\pi x) - 3\sin(\pi x)}{9+\pi^2}$
(B) $y = e^{-3x}\left(\frac{1}{2} + \frac{\pi}{9-\pi^2}\right) + \frac{\pi\cos(\pi x) - 3\sin(\pi x)}{9-\pi^2}$
(C) $y = e^{-3x}\left(\frac{1}{2} + \frac{\pi}{9+\pi^2}\right) + \frac{\pi\cos(\pi x) - 3\sin(\pi x)}{9+\pi^2}$
(D) $y = e^{-3x}\left(\frac{1}{2} + \frac{\pi}{9-\pi^2}\right) - \frac{\pi\cos(\pi x) - 3\sin(\pi x)}{9-\pi^2}$
(E) None of these.

19 (of 25). The arc length of the parametrized curve $x = t^3$, $y = 3t^2/2$ for $1 \leq t \leq 2$ is:

(A) $2\sqrt{2} - 1$ (B) $5\sqrt{5} - 1$ (C) 3 (D) $5\sqrt{5} - 2\sqrt{2}$
(E) None of these.

20 (of 25). The function $f : \mathbb{R} \to \mathbb{R} : x \mapsto x^2 e^{-x^2}$ has

(A) A local minimum at $x = 0$
(B) A point of inflexion at $x = 0$
(C) A local maximum at $x = 0$
(D) A global maximum at $x = 0$
(E) None of these.

21 (of 25). The value of $\lim_{x\to 0} \frac{\sin^2 x}{2x}$ is:

(A) ∞ (B) 1 (C) 0 (D) does not exist
(E) None of these.

22 (of 25). If f is differentiable and strictly positive then $\int f'(x)\sqrt{3f(x)}dx$ is:

(A) $\sqrt{3f(x)} + C$ (B) $f(x)\sqrt{3f(x)} + C$
(C) $\frac{3}{2}\sqrt{f(x)} + C$ (D) $\frac{2\sqrt{3}}{3} f^{3/2}(x) + C$
(E) None of these.

23 (of 25). The Taylor Series for $y = \sin \pi x$ about $x = 0$ is:

(A) $\displaystyle\sum_{k=0}^{\infty} \frac{\pi^{2k} x^{2k}}{(2k+1)!}$ (B) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k \pi^{2k+1} x^{2k+1}}{(2k+1)!}$

(C) $\displaystyle\sum_{k=0}^{\infty} (-\pi)^k \frac{x^{2k}}{(2k+1)!}$ (D) $\displaystyle\sum_{k=0}^{\infty} (-1)^k \frac{(2x)^k}{(2k+1)!\pi^{2k}}$

(E) None of these.

24 (of 25). For what values of $x \in \mathbb{R}$ does the Taylor series of $f(x) = 1/x$ about 2 converge?

(A) $x \in \mathbb{R} \setminus \{0, 2\}$ (B) $|x - 2| < 2$

(C) $|x| < 2$ (D) $|x| < 2$ and $x \neq 0$

(E) None of these.

25 (of 25). Which function has Taylor series

$$1 - 2x + 2x^2 - \frac{4}{3}x^3 + \cdots + (-1)^k \frac{2^k}{k!} x^k + \ldots$$

about $x = 0$?

(A) $\ln(2x)$ (B) $\cos(2x)$ (C) $\sin(2x)$ (D) e^{-2x}

(E) None of these.

End of examination.

1.

1. B	2. B	3. D	4. C	5. A
6. C	7. D	8. D	9. A	10. D
11. E	12. A	13. C	14. D	15.A
16. C	17. C	18. A	19. D	20. A
21. C	22. D	23. B	24. B	25. D